専門科目(TG010001)

流体力学II
Fluid Mechanics II

劉　浩
太田匡則

境界層１
(Boundary Layer)

- 境界層の例：平板間の流れ（クェット、ポアズイユ）

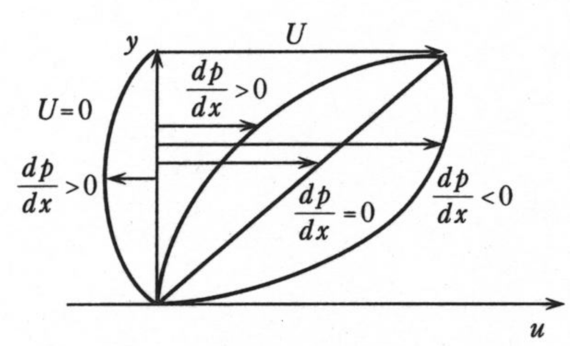
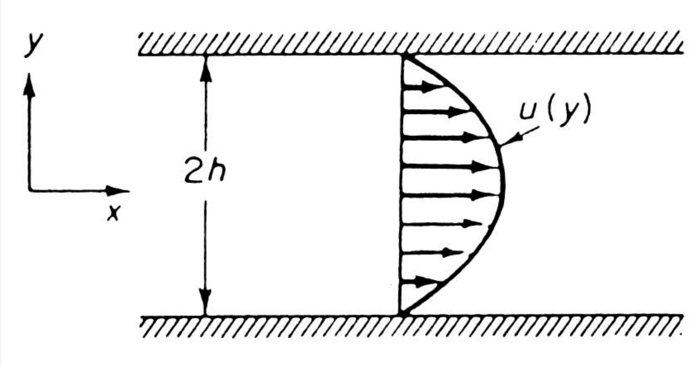


図 5·7　速度分布



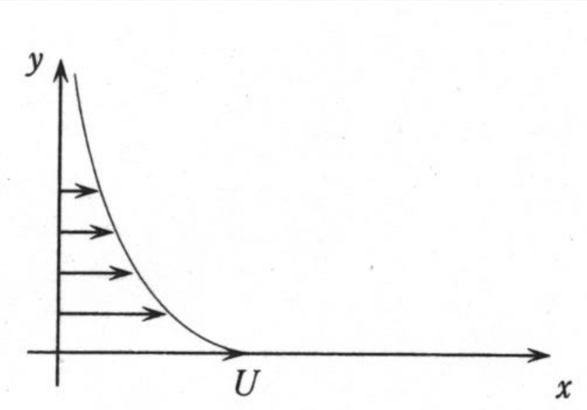


図 5·10　レイリー問題

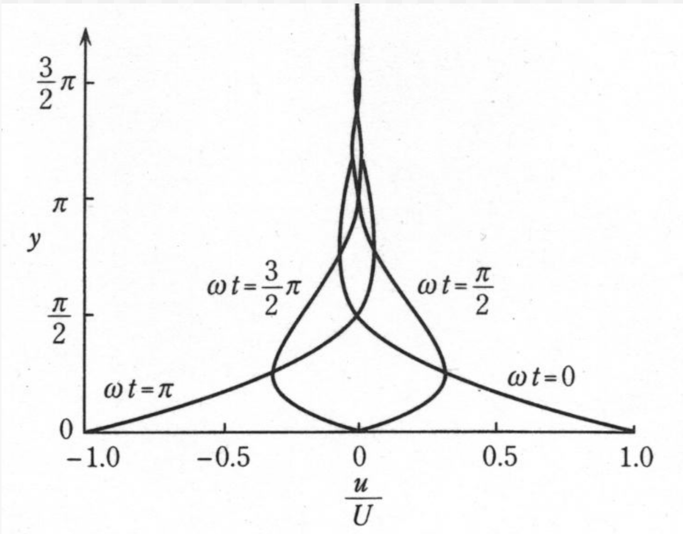
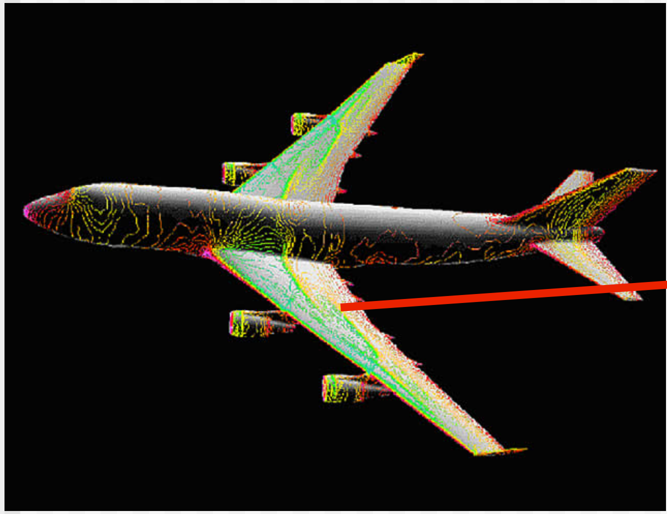
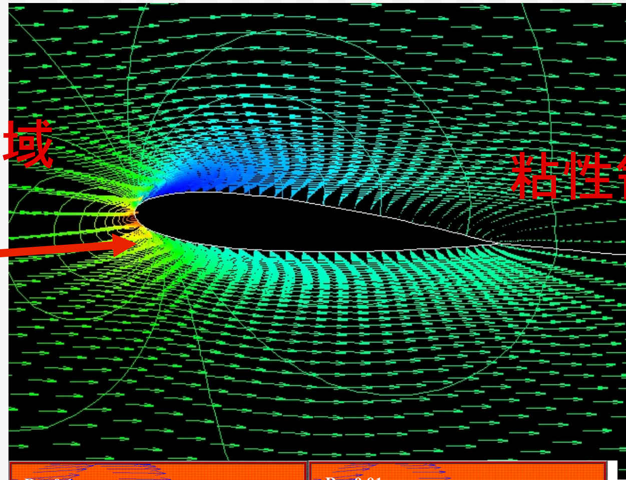


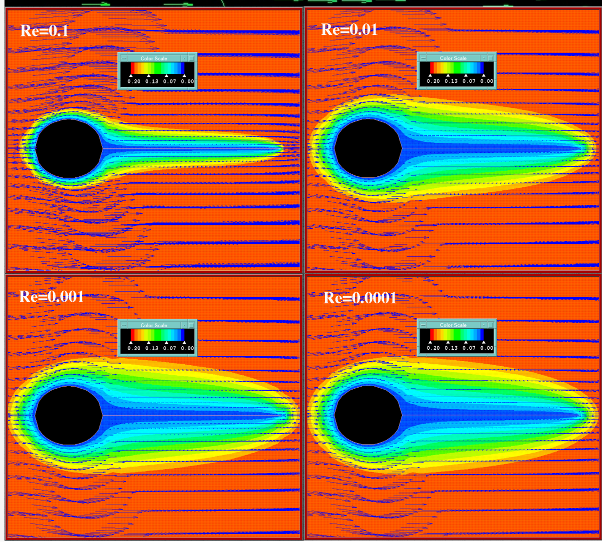
図 5·13　振動平板による流れ

境界層２
(Boundary Layer)

- **境界層の例：工学問題**

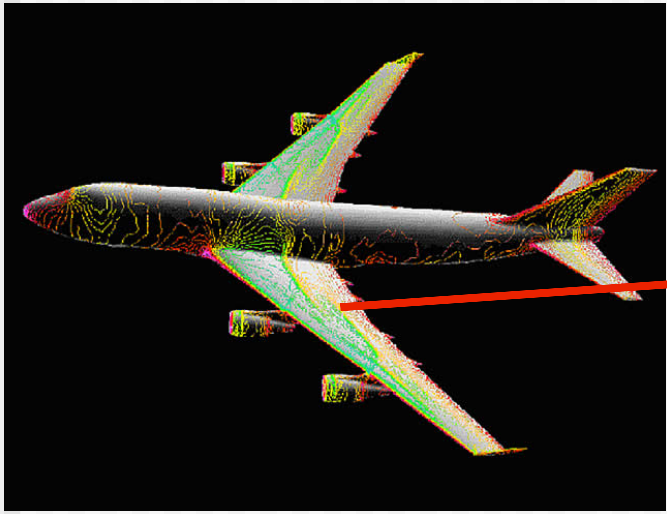




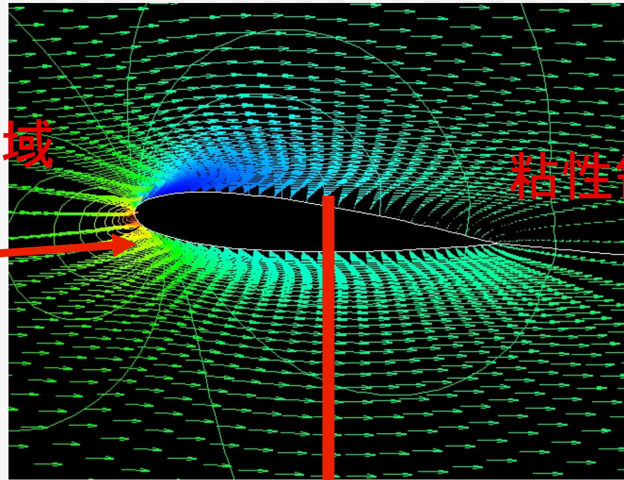


境界層3
(Boundary Layer)

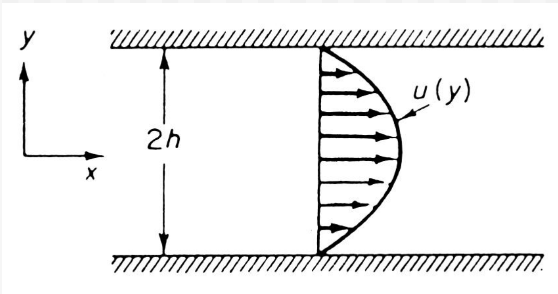
- 境界層の例：工学問題

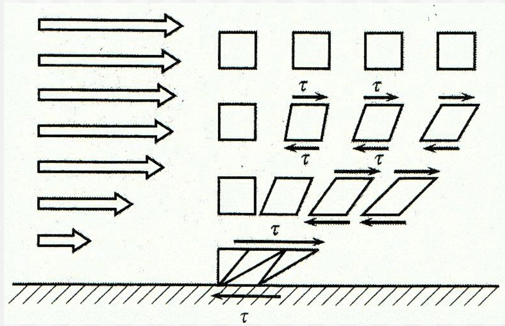


非粘性領域



粘性領域

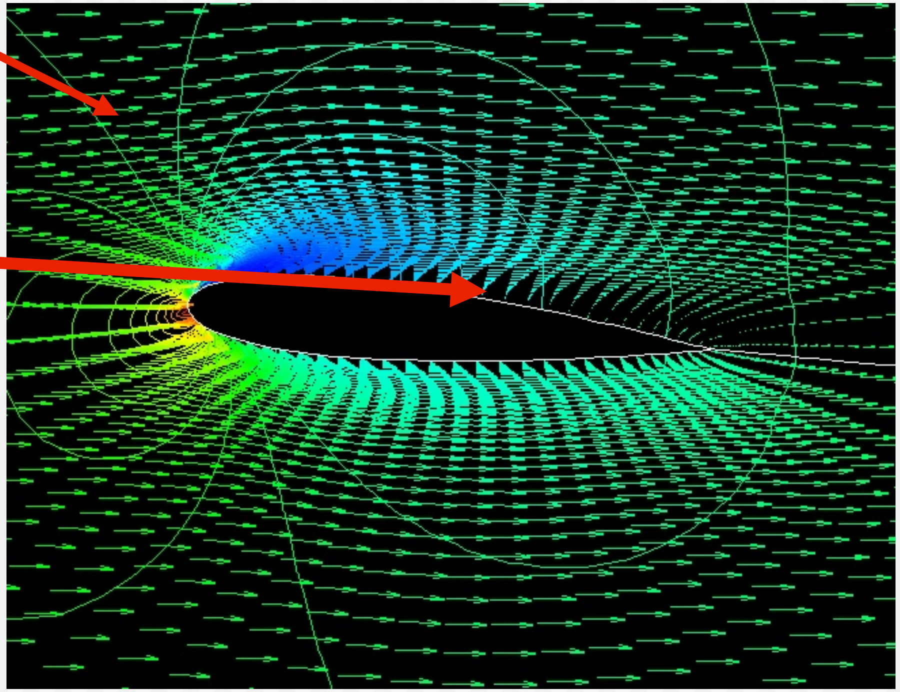




粘性によるせん断応力～変形

境界層方程式 (Boundary Layer Equation)

- 流体支配方程式：
- 境界層近似：
  @物体から離れた所
  @物体近傍



流体の支配方程式 (Governing Equations)

- 連続の式：

$$\frac{\partial\rho}{\partial t} + \rho divdV = 0 \Rightarrow$$

$$\frac{\partial\rho}{\partial t} + \rho(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) = 0$$ 圧縮性流体

$$\frac{\partial\rho}{\partial t} = 0, \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$ 非圧縮性流体

- ナビエ・ストークス方程式：

$$\frac{DV}{Dt} = F - \frac{1}{\rho} gradp + \nu\nabla^2 V$$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + F_x$$

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + F_y$$

プロファイル積分法
(Integration(Karman) Equation of Boundary Layer)

- 境界層内の運動量保存則：

$$BCs: y=0 \rightarrow u=0; y=\delta \rightarrow u=U(x)$$

$$FluxBalance:$$

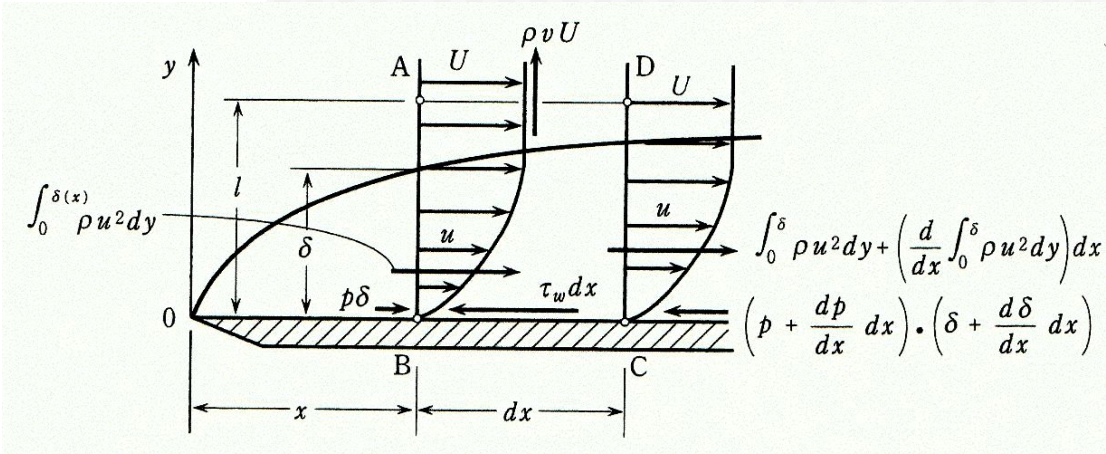
$$AB: \int_o^{\delta(x)} \rho u^2 dy$$

$$CD: \int_o^{\delta(x+\delta x)} [\rho u^2 + \frac{d(\rho u^2)}{dx} dx] dy$$

$$AD: \rho v U dx$$

$$BC: \tau_w dx = \mu \frac{du}{dy} dx$$

$$\frac{d}{dx}\int_o^{\delta}(U-u)u dy - \frac{dU}{dx}\int_o^{\delta} u dy = \nu \frac{\partial u}{\partial y} - \delta U \frac{dU}{dx}$$



境界層近似
(Boundary layer Approximation)

- 境界層近似：

@境界層は薄い、**δ/L=**小さい

$$\delta / l \approx O(0) \rightarrow x \propto l, y \propto \delta$$

@境界層を横切る方向の速度成分**v～δ**

$$v / U \approx O(0) \rightarrow v \propto \delta \Rightarrow u \propto l, v \propto \delta$$

@圧力は境界層内**(y**方向**)**で一定

$$\frac{\partial p}{\partial y} \approx O(0)$$

@粘性項のうち、**x**方向の２回微分は省略できる

$$\frac{\partial u}{\partial x} / \frac{\partial u}{\partial y} \approx O(0) \rightarrow \frac{\partial^2 u}{\partial x^2} \approx O(0)$$

境界層方程式の導出1

(Formation of Boundary Layer Equation)

- 代表的物理量：
  代表長さ＝L　代表速度＝U　代表時間＝T=L/U
- 無次元化:

(u,v,w) = U(u*,v*,w*), (x,y,z) = L(x*,y*,z*), t=t*L/U $\mathrm{Re}=\frac{UL}{\nu}$

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial y}v=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{U}{L/U}\frac{\partial u^*}{\partial t^*}+\frac{U^2}{L}\left(\frac{\partial u^*}{\partial x^*}u^*+\frac{\partial u^*}{\partial y^*}v^*\right)=-\frac{1}{\rho L}\frac{\partial p}{\partial x^*}+\nu\frac{U}{L^2}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$\frac{\partial u^*}{\partial t^*}+\frac{\partial u^*}{\partial x^*}u^*+\frac{\partial u^*}{\partial y^*}v^*=-\frac{\partial p^*}{\partial x^*}+\frac{1}{UL/\nu}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)\Leftarrow p^*=\frac{p}{\rho U^2}$$

↑ $x,u=O(l),\ y,v=O(\delta),\ \frac{1}{\mathrm{Re}}=O(\delta^2)$ ↑

境界層方程式の導出2

(Formation of Boundary Layer Equation)

- **境界層方程式とその特徴：　有次元化**

(u,v,w) = U(u*,v*,w*), (x,y,z) = L(x*,y*,z*), t=t*L/U $\mathrm{Re} = \frac{UL}{\nu}$

$$\frac{\partial u^*}{\partial x^*}u^* + \frac{\partial u^*}{\partial y^*}v^* = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right) \rightarrow \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$p + \rho\frac{U^2}{2} = Const \Rightarrow \frac{dp}{dx} + \rho U\frac{dU}{dx} = 0$$

$$\frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \qquad x,u = O(l),\ y,v = O(\delta),\ \frac{1}{\mathrm{Re}} = O(\delta^2)$$

$$\frac{\partial p}{\partial y} = 0$$

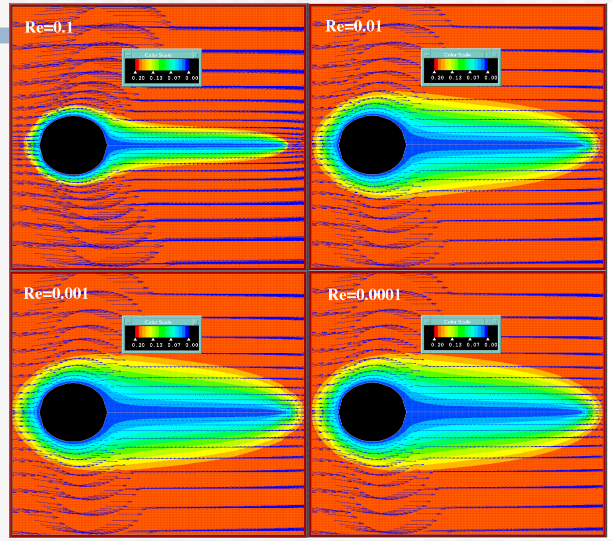
境界層近似の制限

(Limits pf Boundary layer Approximation)

- 境界層近似の制限：

  @Re数が十分小さい、
  粘性境界層が厚い場合

$$\frac{1}{\mathrm{Re}} \neq O(\delta^2)$$

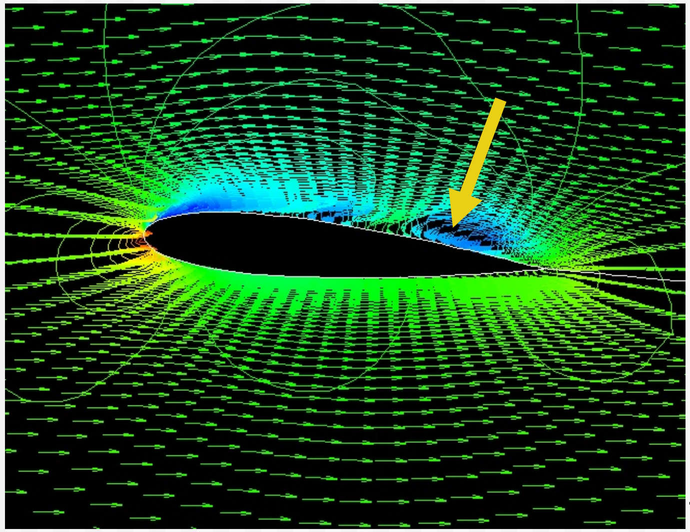


  @平板などの先端付近の流れ
  （x方向変化有）

$$\frac{\partial u}{\partial x} \neq O(\delta)$$

  @表面から流線が離れる剥離点近傍より
  下流などの領域で法線方向速度が
  小さくない場合

$$v \neq \delta$$



ストークス流れ1 (Stokesian Flow)

- 支配方程式：

$$divV = 0 \qquad \frac{\partial V}{\partial t} + V \quad V = F - \frac{1}{\rho} gradp + \nu \nabla^2 V$$

- ストークス近似：移流項を無視
- 一様流れの中に置かれた球まわりの流れ：

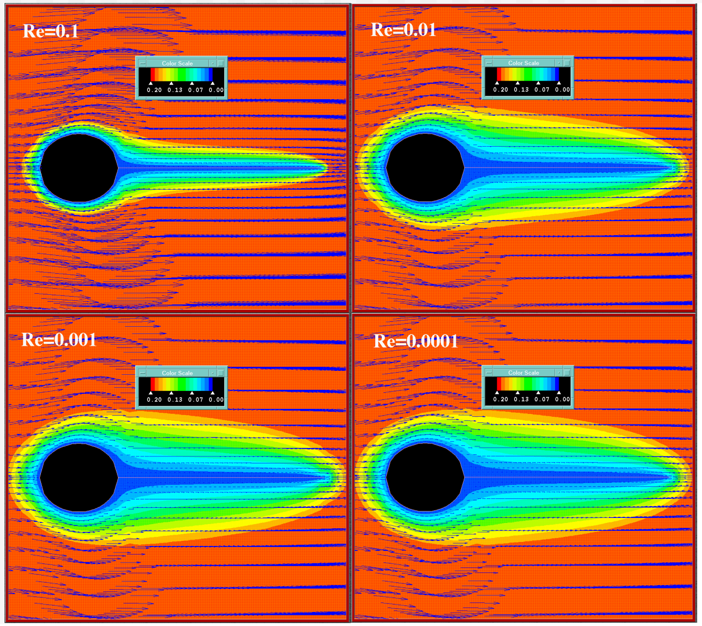
$$BodySurface : u = v = w = 0$$

$$OutsideBoundary : u = U , p = p_\infty$$

$$v_r = U \cos\theta (1 - \frac{3/2 \bullet a}{r} + \frac{1/2 \bullet a^3}{r^3})$$

$$v_\theta = -U \sin\theta (1 - \frac{3/4 \bullet a}{r} - \frac{1/4 \bullet a^3}{r^3})$$

$$p = p_\infty - \frac{3}{2}\mu \frac{U\cos\theta}{a} , \tau_{r\theta} = \frac{3}{2}\mu \frac{U \sin\theta}{a}$$



ストークス流れ2 (Stokesian Flow)

- 抵抗の式：

$$Drag_x = -\int_0^{\pi} \tau_{r\theta} \sin\theta ds - \int_0^{\pi} p \cos\theta ds$$

$$= 4\pi a\mu U + 2\pi a\mu U = 6\pi a\mu U$$

$$C_d = \frac{Drag}{\frac{1}{2}\rho U^2 \pi a^2} = \frac{24}{Re}$$

- ２次元円柱まわりのストークス流れが存在しない
  ストークスのパラドックス

@オゼーン近似：

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} = F - \frac{1}{\rho} gradp + \nu\nabla^2 V$$

→

$$Drag_x = -\int_0^{\pi} \tau_{r\theta} \sin\theta ds - \int_0^{\pi} p \cos\theta ds$$

$$C_d = \frac{Drag}{\frac{1}{2}\rho U^2 \pi a^2} = \frac{24}{Re}(1 + \frac{3}{16}Re)$$